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Boundedness of two-point correlators covariant under the meta-conformal algebra

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Abstract. Covariant two-point functions are derived from Ward identities. For several extensions of dynamical scaling, notably Schrödinger-invariance, conformal Galilei invariance or meta-conformal invariance, the results become unbounded for large time- or space-separations. Standard ortho-conformal invariance does not have this problem. An algebraic procedure is presented which corrects this difficulty for meta-conformal invariance in $(1+1)$ dimensions. A canonical interpretation of meta-conformally covariant two-point functions as correlators follows. Galilei-conformal correlators can be obtained from meta-conformal invariance through a simple contraction. All these two-point functions are bounded at large separations, for sufficiently positive values of the scaling exponents.

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1 Introduction

Dynamical symmetries are powerful tools in investigations of many complex systems. The best-known examples are *conformal invariance* in equilibrium phase transitions [4, 6] and *Schrödinger-invariance* in time-dependent phenomena [8, 12]. One of the most elementary predictions of dynamical symmetries concerns the form of the co-variant two-point functions, to be derived form the (e.g. conformal or Schrödinger) Ward identities, [6, 12]. These are built from quasi-primary scaling operators $\phi_i(t_i, \boldsymbol{r}_i)$, depending locally on a 'time' coordinate $t_i \in \mathbb{R}$ and a 'space' coordinate $\boldsymbol{r}_i \in \mathbb{R}^d$. Since both conformal and Schrödinger groups contain time- and space-translations, and also spatial rotations, we can restrict to the difference $t := t_1 - t_2$ and the absolute value $r := |\boldsymbol{r}| = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$. For conformal [18] and Schrödinger-invariance [8], respectively, the covariant two-point functions of *scalar* quasi-primary operators read (up to a global normalisation constant)

$$C_{12,conf}(t;\boldsymbol{r}) = \langle \phi_1(t,\boldsymbol{r})\phi_2(0,\boldsymbol{0})\rangle = \delta_{x_1,x_2}\left[t^2+r^2\right]^{-x_1} \tag{1}$$

$$R_{12,Schr}(t;\boldsymbol{r}) = \left\langle \phi_1(t,\boldsymbol{r})\widetilde{\phi}_2(0,\boldsymbol{0})\right\rangle = = \delta_{x_1,x_2}\delta(\mathcal{M}_1+\widetilde{\mathcal{M}}_2)\, t^{-x_1}\, \exp\left[-\frac{\mathcal{M}_1}{2}\frac{r^2}{t}\right] \tag{2}$$

The properties of the conformally invariant two-point function are described by the scaling dimensions x_i. It is a *correlator* and is symmetric under permutation of the two scaling operators, viz. $C_{12}(t;\boldsymbol{r}) = C_{21}(-t;-\boldsymbol{r})$. The result (1) is a physically reasonable correlator which decays to zero, if $x_i > 0$, for large time- or space separations, viz. $|t| \to \infty$ or $r \to \infty$.

The Schrödinger-invariant two-point function is a (linear) *response function* – recast here formally as a correlator by appealing to Janssen-de Dominicis theory [21], where $\widetilde{\phi}_i$ is the response operator conjugate to the scaling operator ϕ_i. The two-point function is now characterised by the pair $(x_i, \mathcal{M}_i)$ of a scaling dimension and a mass $\mathcal{M}_i$ associated to each scaling operator ϕ_i. For 'usual' scaling operators, masses are positive by convention, whereas response operators $\widetilde{\phi}_i$ have formally negative masses, viz. $\widetilde{\mathcal{M}}_i = -\mathcal{M}_i < 0$. Because of causality, a response function must vanish for $t < 0$, viz. $R_{12} = 0$ but one has $R_{12} \neq 0$ for $t > 0$; hence it is maximally asymmetric under permutation of the scaling operators.
The result (2) of Schrödinger-invariance does not contain the causality requirement $t > 0$. In addition, it is *not* obvious why the response should vanish for large separations, even if $x_i > 0$ is admitted. Although one might insert these features by hand, it is preferable to derive such conditions formally. One may do so following the procedure [11]:

(i) consider the mass $\mathcal{M}$ as an additional coordinate and dualise by Fourier-transforming with respect to $\mathcal{M}_i$, which introduces dual coordinates ζ_i. The terminology is borrowed from non-relativistic versions of the AdS/CFT correspondence.
(ii) construct an extension of the Schrödinger Lie algebra $\widetilde{sch}(d) := \mathfrak{sch}(d) \oplus \mathbb{C}N$, where the new generator N is in the Cartan sub-algebra of $\widetilde{\mathfrak{sch}}(d)$.
(iii) use the extended Schrödinger Ward identities, in the dual coordinates, to find the co-variant two-point function $\widehat{R}(\zeta_1-\zeta_2, t, \boldsymbol{r})$.
(iv) finally, transform back to the fixed masses $\mathcal{M}_i$. The result is [11, 13]

$$R_{12,\widetilde{Schr}}(t;\boldsymbol{r}) = \delta_{x_1,x_2}\delta(\mathcal{M}_1+\widetilde{\mathcal{M}}_2)\,\Theta(\mathcal{M}_1 t)\, t^{-x_1}\exp\left[-\frac{\mathcal{M}_1}{2}\frac{r^2}{t}\right] \tag{3}$$

With the convention $\mathcal{M}_1 > 0$, the Heaviside function Θ expresses the causality condition $t > 0$. In addition, if $x_i > 0$, the response function decays to zero for large time- or space separations, as physically expected.

A similar problem with boundedness of two-point function arise also for *conformal galilean algebra* [2, 9, 10, 16, 17] $cga(d)$[1]. However, for the $cga(d)$ algebra, it has been shown recently that a procedure analogous to the one of the Schrödinger algebra, as outlined above, can be applied to assure the boundedness of two-pint function which in this case does obey the symmetry relations of a correlator [14].

In this paper[2] we wish to demonstrate that an algebraically sound procedure, as outlined above, to the formulation of Ward identities which physically reasonable results, can be applied to *meta-conformal algebra*. This may not appear obvious, since it is semi-simple, in contrast to Schrödinger and conformal galilean algebra which are not. Our results are stated in Theorems 1 and 2 in section 4.

2 Meta-conformal algebra and two-point function

We shall call *meta-conformal algebra* $\mathfrak{mconf}(d)$ a non-standard representation of conformal algebra which leads to a two-point correlation function distinct from (1). To be precise, we shall distinguish between ortho- and meta-conformal transformations.[3]

Definition 1. *(i)* Meta-conformal transformations *are maps* $(t, r) \mapsto (t', r') = M(t, r)$*, depending analytically on several parameters, such that they form a Lie group. The associated Lie algebra is isomorphic to the conformal Lie algebra* $\mathfrak{conf}(d)$.
(ii) Ortho-conformal transformations *(called 'conformal transformations' for brevity) are those meta-conformal transformations* $(t, r) \mapsto (t', r') = O(t, r)$ *which keep the angles in the time-space of points* $(t, \boldsymbol{r}) \in \mathbb{R}^{1+d}$ *invariant.*

In this paper, we study the meta-conformal transformations, in $(1 + 1)$ time and space dimensions, with the following infinitesimal generators [10, 12]:

$$
\begin{aligned}
X_n &= -t^{n+1}\partial_t - \mu^{-1}[(t+\mu r)^{n+1} - t^{n+1}]\partial_r \\
&\quad -(n+1)\frac{\gamma}{\mu}[(t+\mu r)^n - t^n] - (n+1)xt^n \\
Y_n &= -(t+\mu r)^{n+1}\partial_r - (n+1)\gamma(t+\mu r)^n
\end{aligned}
\tag{4}
$$

such that μ^{-1} can be interpreted as a velocity ('speed of light or sound') and where x, γ are constants (*'scaling dimension'* and *'rapidity'*). The generators obey the Lie algebra, for $n, m \in \mathbb{Z}$

$$
\begin{aligned}
&[X_n, X_m] = (n-m)X_{n+m}, \quad [X_n, Y_m] = (n-m)Y_{n+m} \\
&[Y_n, Y_m] = \mu(n-m)Y_{n+m}
\end{aligned}
\tag{5}
$$

[1] CGA(d) is non-isomorphic to either the standard Galilei algebra or else the Schrödinger algebra $\mathfrak{sch}(d)$. It is a maximal finite-dimensional sub-algebra of non-semi-simple 'altern-Virasoro algebra' $\mathfrak{altv}(1)$ (but without central charges) [3, 7, 9, 11]

[2] Following mainly our original work [20]

[3] From the greek prefixes $o\varrho\theta o$: right, standard; and $\mu\varepsilon\tau\alpha$: of secondary rank.

The isomorphism of (5) with the conformal Lie algebra $\mathfrak{conf}(2)$ is seen for example in [10, 14, 20].

The meta-conformal Lie algebra (5) acts as a dynamical symmetry on the linear advection equation [15]

$$\mathcal{S}\phi(t,r) = (-\mu\partial_t + \partial_r)\phi(t,r) = 0 \tag{6}$$

in the sense that a solution ϕ of $\mathcal{S}\phi = 0$, with scaling dimension $x_\phi = x = \gamma/\mu$, is mapped to another solution of the same equation. This follows from ($n \in \mathbb{Z}$)

$$[\mathcal{S}, Y_n] = 0, \quad [\mathcal{S}, X_n] = -(n+1)t^n\hat{S} + n(n+1)(\mu x - \gamma)t^{n-1} \tag{7}$$

Hence the space of solutions of $\mathcal{S}\phi = 0$ is meta-conformal invariant [10] (extended to Jeans-Poisson systems in [19]).
Now, quasi-primary scaling operators [4] are characterised by the parameters (x_i, γ_i) (μ is simply a global dimensionful scale) and by co-variance under the maximal finite-dimensional sub-algebra $\langle X_{\pm1,0}, Y_{\pm1,0}\rangle \cong \mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$ for $\mu \neq 0$. Explicitly

$$\begin{aligned}
X_{-1} &= -\partial_t \;, \quad X_0 = -t\partial_t - r\partial_r - x \\
X_1 &= -t^2\partial_t - 2tr\partial_r - \mu r^2\partial_r - 2xt - 2\gamma r \\
Y_{-1} &= -\partial_r \;, \quad Y_0 = -t\partial_r - \mu r\partial_r - \gamma \\
Y_1 &= -t^2\partial_r - 2\mu tr\partial_r - \mu^2 r^2\partial_r - 2\gamma t - 2\gamma\mu r
\end{aligned} \tag{8}$$

Here, the generators X_{-1}, Y_{-1} describe time- and space-translations, Y_0 is a (conformal) Galilei transformation, X_0 gives the dynamical scaling $t \mapsto \lambda t$ of $r \mapsto \lambda r$ (with $\lambda \in \mathbb{R}$) such that the so-called 'dynamical exponent' $z = 1$ since both time and space are re-scaled in the same way and finally X_{+1}, Y_{+1} give 'special' meta-conformal transformations.

Using the generators (8) (in their two-body forms $X_n^{[2]}, Y_n^{[2]}$) we construct the meta-conformal Ward identities $X_n^{[2]}\langle\phi_1\phi_2\rangle = Y_n^{[2]}\langle\phi_1\phi_2\rangle = 0$. One obtains the co-variant two-point function, up to normalisation [10, 12]

$$\langle\phi_1(t_1,r_1)\phi_2(t_2,r_2)\rangle = \delta_{x_1,x_2}\delta_{\gamma_1,\gamma_2}\,(t_1-t_2)^{-2x_1}\left(1 + \mu\frac{r_1-r_2}{t_1-t_2}\right)^{-2\gamma_1/\mu} \tag{9}$$

clearly *distinct* from the result (1) of ortho-conformal invariance. However, the result (9) raises immediately the following questions:

1. is $\langle\phi_1\phi_2\rangle$ a correlator or rather a response, since neither of the symmetry or causality conditions are obeyed ?
2. even if $x_i > 0$ and $\gamma_i/\mu > 0$, why does $\langle\phi_1\phi_2\rangle$ not always decay to zero for large separations $|t_1 - t_2| \to \infty$ or $|r_1 - r_2| \to \infty$?
3. why is there a singularity at $\mu(r_1 - r_2) = -(t_1 - t_2)$?

3 Two-point function in dual space

Our construction follows the same steps as outlined above which have already been used to recast the co-variant two-point functions of Schrödinger- and conformal Galilean invariance into a physically reasonable form, see [11, 13, 14].

First, we consider the 'rapidity' γ as a new variable and dualise it through a Fourier transformation, which gives the quasi-primary scaling operator

$$\hat{\phi}(\zeta,t,r) = \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}} \mathrm{d}\gamma\; e^{\mathrm{i}\gamma\zeta}\,\phi_\gamma(t,r) \tag{10}$$

The representation (4) of the meta-conformal algebra becomes

$$\begin{aligned} X_n &= \frac{\mathrm{i}(n+1)}{\mu}\left[(t+\mu r)^n - t^n\right]\partial_\zeta \\ &\quad -t^{n+1}\partial_t - \frac{1}{\mu}\left[(t+\mu r)^{n+1} - t^{n+1}\right]\partial_r - (n+1)xt^n \\ Y_n &= \mathrm{i}(n+1)\,(t+\mu r)^n\,\partial_\zeta - (t+\mu r)^{n+1}\,\partial_r \end{aligned} \tag{11}$$

Second, we seek an extension of the Cartan sub-algebra $\mathfrak{h}$ by looking for a new generator N such that $[X_n,N]=\alpha_n X_n$ and $[Y_m,N]=\beta_m Y_m$ where α_n,β_m are constants to be determined. It turned out [20] that N must have the form

$$N := -r\partial_r - (\zeta+c)\,\partial_\zeta + \mu\partial_\mu - \nu \tag{12}$$

$$[X_n,N]=0\;\;,\;\;[Y_n,N]=-Y_n\;\; n\in\mathbb{Z}. \tag{13}$$

and satisfy (13). N is a dynamical symmetry of (6), since $[\mathcal{S},N]=-\mathcal{S}$. This achieves the construction of the extended meta-conformal algebra $\widetilde{\mathfrak{mconf}}(2) := \mathfrak{mconf}(2)\oplus\mathbb{C}N$, with commutators (5,13).

Third, co-variant two-point functions of quasi-primary scaling operators are found from the Ward identities $X_n^{[2]}\langle\phi_1\phi_2\rangle = Y_n^{[2]}\langle\phi_1\phi_2\rangle = N^{[2]}\langle\phi_1\phi_2\rangle = 0$, with $X_n,Y_n,N\in\widetilde{\mathfrak{mconf}}(2)$ and $n=\pm1,0$ [12]. Given the form of N, we also consider μ to a be further variable and set

$$\langle\hat{\phi}(\zeta_1,t_1,r_1;\mu_1)\hat{\phi}(\zeta_2,t_2,r_2;\mu_2)\rangle = \hat{F}(\zeta_1,\zeta_2,t_1,t_2,r_1,r_2;\mu_1,\mu_2) \tag{14}$$

Clearly, co-variance under X_{-1} and Y_{-1} implements time- and space-translation-invariance, such that $\hat{F}=\hat{F}(\zeta_1,\zeta_2,t,r;\mu_1,\mu_2)$, with $t=t_1-t_2$ and $r=r_1-r_2$. Next, co-variance under other generators of $\widetilde{\mathfrak{mconf}}(2)$ provides the system

$$X_0 : (t\partial_t + r\partial_r + x_1 + x_2)\,\hat{F} = 0 \tag{15}$$
$$Y_0 : (t\partial_r + \mu_1 r\partial_r - \mathrm{i}(\partial_{\zeta_1} + \partial_{\zeta_2}) + (\mu_1 - \mu_2)r_2\partial_r)\,\hat{F} = 0 \tag{16}$$
$$X_1 : (\mathrm{i}r(\partial_{\zeta_1} - \partial_{\zeta_2}) - t(x_1 - x_2))\,\hat{F}(\zeta_1, \zeta_2, t, r; \mu) = 0 \tag{17}$$
$$Y_1 : (t + \mu r)\,(\partial_{\zeta_1} - \partial_{\zeta_2})\,\hat{F}(\zeta_1, \zeta_2, t, r; \mu) = 0 \tag{18}$$
$$N : \big(r\partial_r + (\zeta_+ + c)\partial_{\zeta_+} - \mu\partial_\mu + \nu_1 + \nu_2\big)\,\hat{F}(\zeta_+, t, r; \mu) = 0. \tag{19}$$

Since $\hat{F}$ must not any longer depend explicitly on r_2, (16) shows that $\mu_1 = \mu_2 =: \mu$. Similarly, eq. (18) states that $(\partial_{\zeta_1} - \partial_{\zeta_2})\hat{F} = 0$ such that $\hat{F} = \hat{F}(\zeta_+, t, r; \mu)$, with $\zeta_\pm := \frac{1}{2}(\zeta_1 \pm \zeta_2)$. Then eq. (17) produces $x_1 = x_2$.
The three conditions (15,16,19) (we shall absorb from now on c into a translation of ζ_+) fix the function $\hat{F}(\zeta_+, t, r, ; \mu)$ which depends on three variables and the constant μ, and also on the pairs of constants (x_1, ν_1) and (x_2, ν_2) which characterise the two quasi-primary scaling operators $\hat{\phi}_{1,2}$. Solving eq. (15), it follows that $\hat{F}(\zeta_+, t, r; \mu) = t^{-2x}\hat{f}(u, \zeta_+, \mu)$ with $u = r/t$ and $x = x_1 = x_2$. Changing variables according to $v = \zeta_+ + \mathrm{i}u$ and $\hat{f}(u, \zeta_+, \mu) = \hat{g}(u, v, \mu)$, eqs. (16,19) leads to $\hat{g}(u, v, \mu) = \hat{G}(w, \mu)$ and finally[4]

$$\hat{G}(w, \mu) = \hat{G}_0(\mu) w^{-\nu_1 - \nu_2} = \hat{G}_1(\mu)\left(\zeta_+ + \mathrm{i}\frac{\ln(1 + \mu u)}{\mu}\right)^{-\nu_1 - \nu_2}. \tag{20}$$

Since μ is merely a parameter, $\hat{G}_1(\mu)$ is just a normalisation constant.

Proposition.*The dual two-point function, covariant under the generators* $X_{\pm1,0}, \widetilde{Y_{\pm1,0}}, N$ *of the dual representation (11,12) of the meta-conformal algebra* $\widetilde{\mathfrak{mconf}}(2)$, *is up to normalisation*

$$\begin{aligned}\hat{F}(\zeta_1, \zeta_2, t, r) &= \langle \hat{\phi}_1(t, r, \zeta_1)\hat{\phi}_2(0, 0, \zeta_2)\rangle \\ &= \delta_{x_1, x_2}\,|t|^{-2x_1}\left(\frac{\zeta_1 + \zeta_2}{2} + \mathrm{i}\frac{\ln(1 + \mu r/t)}{\mu}\right)^{-\nu_1 - \nu_2}.\end{aligned} \tag{21}$$

4 Inverse dual transformation

Forth, to un-dualise, we write $\hat{F} = \delta_{x_1, x_2}\,|t|^{-2x_1}\hat{f}_\lambda(\zeta_+)$ such that

$$f_\lambda(\zeta_+) := \hat{f}(\zeta_+ + \mathrm{i}\lambda) = (\zeta_+ + \mathrm{i}\lambda)^{-\nu_1 - \nu_2}, \quad \lambda := \frac{\ln(1 + \mu r/t)}{\mu}. \tag{22}$$

A well-known mathematical result on Fourier analysis on Hardy spaces states there exist the following integral representation ($\hat{\mathcal{F}}_\pm(\gamma_+)$ are square-integrable

[4] for more details see [20]

functions) [1,14,20]

$$\sqrt{2\pi}\,\hat{f}(\zeta_+ + \mathrm{i}\lambda) = \Theta(\lambda)\int_0^\infty \mathrm{d}\gamma_+\, e^{+\mathrm{i}(\zeta_+ +\mathrm{i}\lambda)\gamma_+}\hat{\mathcal{F}}_+(\gamma_+) + \Theta(-\lambda)\int_0^\infty \mathrm{d}\gamma_+\, e^{-\mathrm{i}(\zeta_+ +\mathrm{i}\lambda)\gamma_+}\hat{\mathcal{F}}_-(\gamma_+). \tag{23}$$

The inverse Fourier transformation is found by distinguishing the cases $\lambda > 0$ and $\lambda < 0$. In the case $\lambda > 0$, we have from (23)

$$\begin{aligned} F &= \frac{|t|^{-2x}}{\pi\sqrt{2\pi}}\hat{G}_1(\mu)\int_{\mathbb{R}^2}\mathrm{d}\zeta_+\mathrm{d}\zeta_-\ \mathrm{e}^{-\mathrm{i}(\gamma_1+\gamma_2)\zeta_+}\ \mathrm{e}^{-\mathrm{i}(\gamma_1-\gamma_2)\zeta_-}\ \times \\ &\quad\times\int_{\mathbb{R}}\mathrm{d}\gamma_+\,\Theta(\gamma_+)\hat{\mathcal{F}}_+(\gamma_+)\mathrm{e}^{-\gamma_+\lambda}e^{\mathrm{i}\gamma_+\zeta_+} \\ &= \frac{|t|^{-2x}}{\pi\sqrt{2\pi}}\hat{G}_1(\mu)\int_{\mathbb{R}}\mathrm{d}\gamma_+\,\Theta(\gamma_+)\hat{\mathcal{F}}(\gamma_+)e^{-\gamma_+\lambda}\ \times \\ &\quad\times\int_{\mathbb{R}}\mathrm{d}\zeta_-\ e^{-(\gamma_1-\gamma_2)\zeta_-}\int_{\mathbb{R}}\mathrm{d}\zeta_+\ e^{\mathrm{i}(\gamma_+-\gamma_1-\gamma_2)\zeta_+} \\ &= \delta_{x_1,x_2}\delta(\gamma_1-\gamma_2)\Theta(\gamma_1)|t|^{-2x_1}f_1(\mu)f_2(\gamma_1)\exp\left(-2\gamma_1\ln(1+\mu r/t)/\mu\right) \\ &= \delta_{x_1,x_2}\delta(\gamma_1-\gamma_2)\Theta(\gamma_1)f_1(\mu)f_2(\gamma_1)|t|^{-2x_1}(1+\mu r/t)^{-2\gamma_1/\mu}. \end{aligned} \tag{24}$$

where in the third line two delta functions where recognised, and f_1, f_2 contain unspecified dependencies on μ and γ_1, respectively.[5] In the case $\lambda < 0$, we have in quite an analogous way

$$\begin{aligned} F &= \delta_{x_1,x_2}\delta(\gamma_1-\gamma_2)\Theta(-\gamma_1)|t|^{-2x_1}f_1(\mu)f_2(-\gamma_1)\exp\left(-2\gamma_1\ln(1+\mu r/t)/\mu\right) \\ &= \delta_{x_1,x_2}\delta(\gamma_1-\gamma_2)\Theta(-\gamma_1)f_1(\mu)f_2(-\gamma_1)|t|^{-2x_1}(1+\mu r/t)^{-2\gamma_1/\mu}. \end{aligned} \tag{25}$$

The meaning of the signs of λ, is carefully explained in [20]. Under *convention that* $\mu > 0$ we have always $\gamma_1 r/t = |\gamma_1 r/t| > 0$, independently of the sign of λ. Therefore, we can always write for the time-space argument

$$\mu\frac{r}{t} = \frac{\mu}{\gamma_1}\frac{\gamma_1 r}{t} = \frac{\mu}{\gamma_1}\left|\frac{\gamma_1 r}{t}\right| \tag{26}$$

(if $\gamma_1 \neq 0$) and we have identified the source of the non-analyticity in the two-point function. Eqs. (24,25,26) combine to give our main result.

Theorem 1. *With the convention that* $\mu = \mu_1 = \mu_2 > 0$*, and if* $\nu_1+\nu_2 > \frac{1}{2}$*, the two-point correlator, co-variant under the representation (8), enhanced by (12), of the extended meta-conformal algebra* $\widetilde{\mathfrak{conf}}(2)$*, reads up to normalisation*

$$C_{12}(t,r) = \langle\phi_1(t,r)\phi_2(0,0)\rangle = \delta_{x_1,x_2}\delta_{\gamma_1,\gamma_2}\,|t|^{-2x_1}\left(1+\frac{\mu}{\gamma_1}\left|\frac{\gamma_1 r}{t}\right|\right)^{-2\gamma_1/\mu}. \tag{27}$$

[5] An eventual shift $\zeta_+ \mapsto \zeta_+ + c$, see (19), can be absorbed into the re-definition $\hat{\mathcal{F}}(\gamma_+)\,e^{-\gamma_+ c} \mapsto \hat{\mathcal{F}}(\gamma_+)$.

This form has the correct symmetry $C_{12}(t,r) = C_{21}(-t,-r)$ under permutation of the scaling operators of a correlator. For $\gamma_1 > 0$ and $x_1 > 0$, the correlator decays to zero for $t \to \pm\infty$ or $r \to \pm\infty$.

In the limit $\mu \to 0$, the extended meta-conformal algebra (5,13) contracts to the extended altern-Virasoro algebra $\widetilde{\mathfrak{altv}}(1)$, whose maximal finite-dimensional sub-algebra is the extended conformal Galilean algebra $\widetilde{\mathrm{CGA}}(1) = \mathrm{CGA}(1) \oplus \mathbb{C}N$. We recover as a special limit case:

Theorem 2. [14] *If $\nu_1 + \nu_2 > \frac{1}{2}$, the two-point correlator, co-variant under the extended conformal Galilean algebra $\widetilde{\mathrm{CGA}}(1)$, reads (up to normalisation)*

$$C_{12}(t,r) = \langle \phi_1(t,r)\phi_2(0,0)\rangle = \delta_{x_1,x_2}\delta_{\gamma_1,\gamma_2}\,|t|^{-2x_1}\exp\left(-\left|\frac{2\gamma_1 r}{t}\right|\right). \quad (28)$$

Any treatment of the CGA which neglects this non-analyticity cannot be correct.

5 Conclusion

Summarising, we have shown that for time-space meta-conformal invariance, as well as for its $\mu \to 0$ limit conformal galilean invariance (or BMS-invariance), the co-variant two-point correlators are given by eqs. (27,28) and are explicitly *non-analytic* in the temporal-spatial variables. Any form of the Ward identities which implicitly assumes such an analyticity cannot be correct. In our construction of physically sensible Ward identities, we extended the Cartan sub-algebra to a higher rank. The extra generator N provides an important ingredient in the demonstration that in direct space, the meta-conformally and galilean-conformallly covariant two-point correlators rather are *distribution*s.

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